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Linear Inequalities

TOPIC 1 Solution of Linear Inequality and System of Linear Inequalities, Representation of Solution of Linear Inequality in One Variable on a Number Line, Representation of Solution of a Linear Inequality and System of Linear Inequalities in a Cartesian Plane, Equations and Inequations Involving Absolute Value Functions, Greatest Integer Functions, Logarithmic Functions

1. The region represented by $\{z = x + iy \in C : |z| - \text{Re}(z) \le 1\}$ is also given by the inequality: **[Sep. 06, 2020 (I)]**

(a) $y^2 \ge 2(x+1)$ (b) $y^2 \le 2\left(x+\frac{1}{2}\right)$

(c) $y^2 \le x+\frac{1}{2}$ (d) $y^2 \ge x+1$

2. Consider the two sets :

$A = \{m \in \mathbf{R}$: both the roots of $x^2 - (m+1)x + m + 4 = 0$ are real$\}$ and $B = [-3, 5)$.

Which of the following is **not** true? **[Sep. 03, 2020 (I)]**

(a) $A - B = (-\infty, -3) \cup (5, \infty)$

(b) $A \cap B = \{-3\}$

(c) $B - A = (-3, 5)$

(d) $A \cup B = \mathbf{R}$

3. If $A = \{x \in R : |x| < 2\}$ and $B = \{x \in R : |x-2| \ge 3\}$; then : **[Jan. 9, 2020 (II)]**

(a) $A \cap B = (-2, -1)$ (b) $B - A = R - (-2, 5)$

(c) $A \cup B = R - (2, 5)$ (d) $A - B = [-1, 2)$

4. Let S be the set of all real roots of the equation, $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S: **[Jan. 8, 2020 (II)]**

(a) contains exactly two elements.

(b) is a singleton.

(c) is an empty set.

(d) contains at least four elements.

5. All the pairs (x,y) that satisfy the inequality $2\sqrt{\sin^2 x - 2\sin x + 5}\cdot\frac{1}{4\sin^2 y} \le 1$ also satisfy the equation: **[April 10, 2019 (I)]**

(a) $2|\sin x| = 3\sin y$ (b) $2\sin x = \sin y$

(c) $\sin x = 2\sin y$ (d) $\sin x = |\sin y|$

6. The number of integral values of m for which the equation $(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$ has no real root is : **[April 08, 2019 (II)]**

(a) 1 (b) 2

(c) infinitely many (d) 3

7. The number of integral values of m for which the quadratic expression, $(1+2m)x^2 - 2(1+3m)x + 4(1+m)$, $x \in R$, is always positive, is : **[Jan. 12, 2019 (II)]**

(a) 3 (b) 8

(c) 7 (d) 6

8. If $f(x) = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$, $x \in R$, then the equation $f(x) = 0$ has : **[Online April 9, 2014]**

(a) no solution

(b) one solution

(c) two solutions

(d) more than two solutions

9. If a, b, c are distinct +ve real numbers and $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ is **[2002]**

(a) less than 1 (b) equal to 1

(c) greater than 1 (d) any real no.

Hints & Solutions

1. **(b)** $\because |z| - \text{Re}(z) \le 1 \qquad (\because z = x + iy)$

$\Rightarrow \sqrt{x^2+y^2} - x \le 1 \Rightarrow \sqrt{x^2+y^2} \le 1 + x$

$\Rightarrow x^2 + y^2 \le 1 + x^2 + 2x$

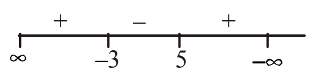
$\Rightarrow y^2 \le 1 + 2x \Rightarrow y^2 \le 2\left(x + \frac{1}{2}\right)$

2. **(a)** $A = \{m \in \mathbf{R} : x^2 - (m+1)x + m + 4 = 0$ has real roots$\}$

$D \ge 0$

$\Rightarrow (m+1)^2 - 4(m+4) \ge 0$

$\Rightarrow m^2 - 2m - 15 \ge 0$



$A = \{(-\infty, -3] \cup [5, \infty)\}$

$B = [-3, 5) \Rightarrow A - B = (-\infty, -3) \cup [5, \infty)$

3. **(b)** $A = \{x : x \in (-2, 2)\}$

$B = \{x : x \in (-\infty, -1] \cup [5, \infty)\}$

$A \cap B = \{x : x \in (-2, -1]\}$

$A \cup B = \{x : x \in (-\infty, 2) \cup [5, \infty)\}$

$A - B = \{x : x \in (-1, 2)\}$

$B - A = \{x : x \in (-\infty, -2] \cup [5, \infty)\}$

4. **(b)** Let $3^x = y$

$\therefore \quad y(y-1) + 2 = |y-1| + |y-2|$

Case 1: when $y > 2$

$y^2 - y + 2 = y - 1 + y - 2$

$y^2 - 3y + 5 = 0$

$\because \quad D < 0$ [$\therefore$ Equation not satisfy.]

Case 2: when $1 \le y \le 2$

$y^2 - y^2 + 2 = y - 1 - y + 2$

$y^2 - y + 1 = 0$

$\because \quad D < 0$ [$\therefore$ Equation not satisfy.]

Case 3: when $y \le 1$

$y^2 - y + 2 = -y + 1 - y + 2$

$y^2 + y - 1 = 0$

$\therefore y = \frac{-1+\sqrt{5}}{2}$

$= \frac{-1-\sqrt{5}}{2}$ [$\therefore$ Equation not Satisfy]

$\therefore$ Only one $-1 + \frac{\sqrt{5}}{2}$ satisfy equation

5. **(d)** Given inequality is,

$2\sqrt{\sin^2 x - 2\sin x + 5} \le 2^{2\sin^2 y}$

$\Rightarrow \sqrt{\sin^2 x - 2\sin x + 5} \le 2\sin^2 y$

$\Rightarrow \sqrt{(\sin x - 1)^2 + 4} \le 2\sin^2 y$

It is true if $\sin x = 1$ and $|\sin y| = 1$

Therefore, $\sin x = |\sin y|$

6. **(c)** Given equation is

$(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$

$\because$ equation has no real solution

$\therefore \; D < 0$

$\Rightarrow 4(1+3m)^2 < 4(1+m^2)(1+8m)$

$\Rightarrow 1 + 9m^2 + 6m < 1 + 8m + m^2 + 8m^3$

$\Rightarrow 8m^3 - 8m^2 + 2m > 0$

$\Rightarrow 2m(4m^2 - 4m + 1) > 0 \Rightarrow 2m(2m-1)^2 > 0$

$\Rightarrow m > 0$ and $m \ne \frac{1}{2}$ $\left[\because \frac{1}{2} \text{ is not an integer}\right]$

$\Rightarrow$ number of integral values of m are infinitely many.

7. **(3)** Let the given quadratic expression

$(1+2m)x^2 - 2(1+3m)x + 4(1+m)$, is positive for all $x \in R$, then

$1 + 2m > 0$ ...(i)

$D < 0$

$\Rightarrow \quad 4(1+3m)^2 - 4(1+2m)4(1+m) < 0$

$\Rightarrow \quad 1 + 9m^2 + 6m - 4[1 + 2m^2 + 3m] < 0$

$\Rightarrow \quad m^2 - 6m - 3 < 0$

$\Rightarrow \quad m \in (3 - 2\sqrt{3}, 3 + 2\sqrt{3})$

From (i)

$\therefore \quad m > -\frac{1}{2}$

$m \in \left(3 - 2\sqrt{3}, 3 + 2\sqrt{3}\right)$

Then, integral values of $m = \{0, 1, 2, 3, 4, 5, 6\}$

Hence, number of integral values of $m = 7$

8. **(b)** $f(x) = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$

Put $f(x) = 0$

$\Rightarrow \quad 0 = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$

$\Rightarrow \quad \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x = 1$

$\Rightarrow \quad 3^x + 4^x = 5^x$ ...(i)

For $x = 1$

$3^1 + 4^1 > 5^1$

For $x = 3$

$3^3 + 4^3 = 91 < 5^3$

Only for $x = 2$, equation (i) Satisfy

So, only one solution $(x = 2)$

9. **(a)** $\because (a-b)^2 + (b-c)^2 + (c-a)^2 > 0$

$\Rightarrow 2(a^2 + b^2 + c^2 - ab - bc - ca) > 0$

$[\because a^2 + b^2 + c^2 = 1]$

$\Rightarrow 2 > 2(ab + bc + ca) \Rightarrow ab + bc + ca < 1$